# RELATIVE NAVIGATION ALGORITHMS FOR PHASE 1 OF THE MMS FORMATION

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**ABSTRACT**

This paper evaluates several navigation approaches for the first phase of the Magnetospheric Multiscale (MMS) mission, which consists of a tetrahedral formation of four satellites in highly eccentric Earth orbits of approximately 1.2 by 12 Earth radii at an inclination of 10º. The inter-satellite separation is approximately 10 kilometers near apogees. Navigation approaches were studied using ground station range and two-way Doppler measurements, Global Positioning System (GPS) pseudorange measurements, crosslink range measurements among the members flying in formation, and various combinations of these measurement types. An absolute position accuracy of 10 kilometers or better can be achieved with most of the approaches studied and a relative position accuracy of 100 meters or better can be achieved at apogee in some cases. Among the various approaches studied, the approaches that use a combination of GPS and crosslink measurements were found to be more reliable in terms of absolute and relative navigation accuracies and operational flexibility.

## 1 – INTRODUCTION

The nominal Magnetospheric Multiscale (MMS) mission has an operational duration of two years. The MMS mission will be conducted in four distinct phases: two equatorial orbit phases (Phases 1-2), one double lunar swingby phase, and one polar orbit phase. At the end of Phase 1, a series of apogee-raising maneuvers will be performed to increase the apogee from 12 to 30 Earth radii ($R_e$). After the Phase 2 mission is concluded, another apogee-raising maneuver will be performed to raise the apogee to lunar distance, approximately 62 $R_e$, which defines the beginning of Phase 3. Phase 3 is the transfer orbit from the final Phase 2 orbital state to the Phase 4 orbital state with perigee of 10 $R_e$, apogee of 50 $R_e$ and 90-degree inclination (with respect to the ecliptic plane) (ref. 1). The Phase 1 mission, which will last for 12 months, consists of four spacecraft flying in a tetrahedral formation with inter-spacecraft separations of approximately 10 kilometers near the apogees. The Phase 1 orbits are 1.2 by 12 $R_e$ orbits at a 10-degree inclination, with a period of approximately one day. Satellites 1, 2, and 3 are in nearly the same orbit plane, separated primarily in the along-track and radial directions. Satellite 4 is separated from these satellites primarily in the cross-track direction. Figure 1 shows the variation of inter-satellite separations computed using the MMS Phase 1 epoch state vectors used for the current investigation, with the minimum separations occurring at apogees and the maximums occurring at perigees.

This paper presents the results of a study performed using the Global Positioning System (GPS) Enhanced Onboard Navigation System (GEONS) to identify approaches that will provide the navigation accuracy needed to meet the science objectives for MMS Phase 1 mission (ref. 2). The science-derived accuracy requirements are (1) a post-processing knowledge of the spacecraft position to within 100 kilometers and (2) knowledge of the inter-spacecraft distances to within 1% of the actual separation near apogee (100 meters for 10-kilometer separations). The navigation accuracy requirements associated with satisfying the MMS Phase 1 formation control objectives are addressed to some extent in this paper and will be evaluated further in a follow-on study.

Both ground-based and onboard navigation concepts were investigated. For a ground-based orbit determination (OD) system, two-way ground station (GS) Doppler and range measurements are provided by the ground tracking network. Processing tracking data acquired onboard, such as crosslink range data, in the ground-based solutions would require downlinking of this data to the ground system. If navigation data is needed onboard, a state vector would be periodically uplinked to the spacecraft and propagated onboard between updates from the ground. If real-time or near real-time OD solutions are needed onboard, the preferred approach is to perform OD onboard by processing GPS pseudorange measurements or a combination of GPS pseudorange and crosslink range measurements. Another approach is to perform OD onboard using only crosslink range measurements with periodic uplinks of absolute ground OD solutions.

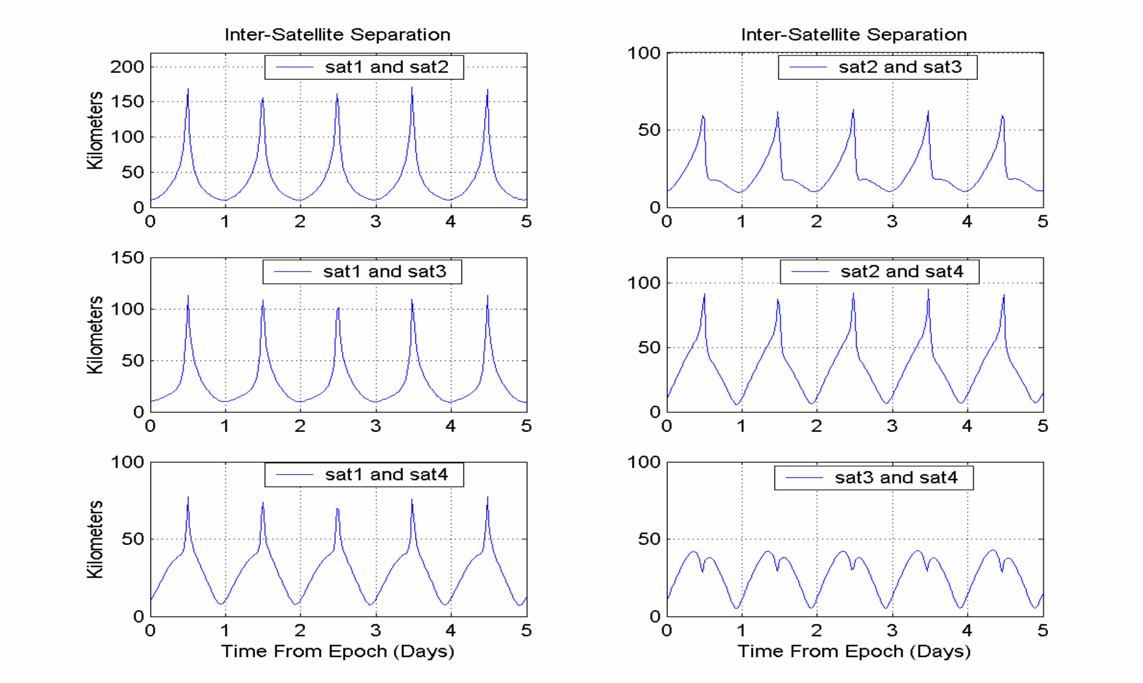


**Figure 1. MMS Phase 1 Inter-Satellite Separations**

The following tracking measurement types were simulated to evaluate the MMS Phase 1 navigation performance:

- GS two-way Doppler and range data
- Two-way crosslink range data (between all pairs of satellites in the formation)
- GPS pseudorange data (with different receiver signal acquisition thresholds)

GEONS (ref. 3) navigation solutions were obtained processing different combinations of these tracking measurement types and evaluated in terms of absolute and relative state errors.

Section 2 of this paper presents the orbit determination accuracy results derived from processing GS Doppler, GS range and crosslink range measurements. Section 3 discusses the results obtained processing GPS pseudorange and crosslink measurements. Section 4 describes the Monte-Carlo simulation results for three selected navigation scenarios. Section 5 summarizes the conclusions.

## 2 – GROUND SOLUTIONS USING GS AND CROSSLINK MEASUREMENTS

This section summarizes MMS Phase 1 navigation solutions obtained processing GS Doppler, GS range and crosslink range measurements. Key parameters used in the measurement simulation and GEONS filter are listed in Tables 1 and 2, respectively.

### 2.1 – Preliminary Analysis of GS Two-Way Doppler Tracking Scenarios

GS S-Band Doppler measurements were simulated using a measurement simulation program (ref. 4) for three Universal Space Network (USN) S-Band tracking stations located at Wallops Island, Hawaii, and Madrid, Spain. An earlier batch OD covariance analysis study (ref. 5) examined a case of GS Doppler-only solutions using three 10-minute contacts per orbit and 3-sigma Doppler measurements noise of 7 milliHertz-S-Band (mHz-S). The covariance analysis showed that 4- to 5-day OD arcs with three 10-minute passes per orbit can give definitive absolute solutions accurate to within 3 kilometers.

**Table 1. Measurement Simulation Parameters**

| Parameter | Nominal Values | Variations |
|---|---|---|
| Measurement noise (1-sigma) and biases : | | |
| GS Doppler random noise | 7 mHz-S | 35 mHz-S |
| GS Doppler bias | 0.0 | |
| GS range random noise | 2 meters | |
| GS range bias | 0.0 | 15 meters |
| Crosslink range random noise | 2 meters | 32, 50 and 100 meters |
| Crosslink range bias | 0.0 | |
| GPS pseudorange random noise | 2 meters for signals for ≥28 dB-Hz<br>100 meters for 20 dB-Hz signal | |
| GPS SV ephemeris and time errors | 2 meters | |
| Allan variance parameters for the GPS receiver clocks | Rubidium (Rb) standard (USO):<br>$h_0$ (second)= $2\times10^{-20}$<br>$h_{-2}$ (1/second)=$4\times10^{-29}$ | Temperature Controlled Crystal Oscillator (TCXO): $h_0$ (second)=$8\times10^{-20}$<br>$h_{-2}$ (1/second)=$4\times10^{-23}$ |
| Atmospheric Delays: | | |
| Ionospheric and tropospheric effects for GS measurements | 100% included with 15-degree elevation angle mask | |
| Ionospheric effects for GPS pseudorange | Signals included with height-of-ray-path greater than 500 kilometers | |
| Measurement rate and pass length: | | |
| GS range and Doppler data | One measurement every 10 seconds<br>Three 20-minute contacts per orbit | Three 10-, 20-, 30-, 60-minute contacts per orbit |
| Crosslink range data (when combined with GS data) | One measurement every 2 minutes between each pair of satellites (continuous tracking) | One measurement every 10 seconds for 2 hours around apogee; one every 6 minutes (continuous tracking) |
| Crosslink range data (when combined with GPS data) | 10 minutes every hour at a rate of one measurement per minute between each pair of satellites | |
| GPS pseudorange data | All-in-view at 1-minute intervals using 31 dB-Hz acquisition threshold | 20, 28, 30, 33, 35dB-Hz acquisition thresholds |

**Table 2. GEONS Processing Parameters**

| Parameter | Values |
|---|---|
| Measurement noise root-variance: | |
| GS (Doppler, range) | (0.05 Hz-S, 10 meters) |
| Crosslink range | 500 meters |
| GPS pseudorange | 25 meters |
| Nonspherical Earth gravity model | 4x4 Joint Gravity Model (JGM)-2 for GS Doppler solutions<br>8x8 JGM-2 for GPS solutions and Monte Carlo simulation |
| Solar and planetary ephemeris | High-precision analytical ephemeris for Sun, Moon, Mars, Venus, Saturn, and Jupiter |
| Estimated state | • Spacecraft position and velocity [in absolute mode]<br>• GPS receiver clock bias and drift when GPS measurements used |
| Initial state errors: | |
| Position/velocity (each component) | 10 kilometers and 0.1 meters/second |
| Drag coefficient | 10% |
| Solar radiation pressure coefficient | 5 - 15 % |
| User clock bias | 100 – 200 meters |
| User clock drift | 0.1 – 0.2 meters/second |
| Initial state root variance: | |
| Position (each component) | 100 kilometers |
| Velocity (each component) | 1 meters/second |
| User clock (bias, drift) | (100 meters, 1 meters/second) |
| State process noise rates: | |
| Velocity (each component) | |
| GS measurements processing | $(0.1, 1.0, 1.0)\times10^{-8}$ |
| GPS measurements processing | $(0.1, 1.0, 1.0)\times10^{-10}$ |
| User clock (bias, drift) | $(1\times10^{-6}, 1\times10^{-6})$ (Rb Clock) |

The GEONS OD simulation study also used three tracking contacts for each satellite per orbit, one from each GS. However, three 10-minute contacts per orbit were found to give somewhat unstable solutions. A preliminary GS Doppler tracking scenario analysis was performed to find a baseline GS Doppler tracking scenario that gives more stable solutions. Four cases were studied using three 10-, 20-, 30-, and 60-minute contacts per orbit. It was also assumed that the satellites in the formation were tracked sequentially with 10-, 20- or 30-minute gaps between consecutive satellite contacts. The GS Doppler noise assumed for this preliminary tracking scenario analysis was 7 mHz-S (1-sigma), which is 3 times the Doppler noise level used for the batch OD error analysis mentioned above. For MMS Phase 1, the GSs could be from the USN, which generally gives higher Doppler noise values than DSN stations. A case with a higher Doppler noise level of 35 mHz-S (1-sigma) was also studied to assess the navigation solution sensitivity to the GS Doppler noise level.

In this preliminary GS Doppler tracking analysis, only the first two satellites in the formation are considered. The same tracking pattern will repeat three times every orbit (about 1 day in the case of MMS Phase 1) using different GSs. Time gaps between two tracking passes by the same GS will be determined primarily by the GS operational constraints. From a relative OD point of view, however, smaller gaps are preferred, because that might lead to more cancellation among errors from error sources common to spacecraft flying in close proximity and would produce smaller relative errors.

Table 3 summarizes the root-mean-square (RMS) and maximum position errors associated with solutions for the above four cases. The filter convergence was reached in less than 2 days.

**Table 3. Steady State Absolute and Relative Position Errors Obtained Using GS Doppler Data**

| Case | Contact Length/gap (minute) | Satellite | Absolute Position Errors (km) | | Relative Position Errors (km) | |
|---|---|---|---|---|---|---|
| | | | RMS | Maximum | RMS | Maximum |
| 1 | 10/20 | Sat1 | 1.44 | 2.53 | 2.17 | 3.90 |
| | | Sat2 | 0.94 | 2.57 | | |
| 2 | 20/10 | Sat1 | 1.11 | 2.21 | 3.15 | 5.79 |
| | | Sat2 | 2.20 | 4.40 | | |
| 3 | 30/30 | Sat1 | 0.87 | 2.11 | 1.91 | 3.27 |
| | | Sat2 | 1.10 | 1.80 | | |
| 4 | 60/30 | Sat1 | 0.88 | 1.68 | 2.01 | 3.06 |
| | | Sat2 | 1.22 | 1.84 | | |

The results of these preliminary solutions show that the maximum absolute position errors are typically a few kilometers, and the relative errors are generally larger than the absolute errors, indicating that there is little or no cancellation of errors between satellites 1 and 2. The Doppler-only solutions presented here also indicate that larger relative position errors of 3- to 4-kilometers appear near apogee. Science requires relative position errors of 100 meters or less near the Phase 1 apogee. These preliminary results show that it is not feasible to meet this relative accuracy goal using GS Doppler-only solutions.

These results suggest that, to achieve the science relative position accuracy goal, additional tracking measurement types are needed. Although the Case 2 solutions has larger errors (see Table 3) than Case 1 in this preliminary tracking data analysis with satellites 1 and 2, this is the case that gives more stable solutions, especially when combined with other data types. It was observed that solutions obtained using longer tracking contacts were generally more stable in terms of filter convergence. Therefore, the Case 2 tracking scenario with three 20-minute GS tracking contacts and a 10-minute gap was used as baseline for all solutions to be studied using combinations of the GS data with other tracking measurements. This choice constitutes a "minimum" GS Doppler tracking scenario that would give stable filter solutions.

### 2.2 - Solutions Using GS Doppler, GS Range, and Crosslink Range Measurements

Measurements used for the solutions were simulated using the nominal parameter values listed in Table 1. The GS range data has the same data rate and tracking schedule as the corresponding Doppler measurements. The crosslink range measurements are two-way without any range bias and remote-to-remote as well as local-to-remote crosslinks are included. In the case of the crosslink range measurements, the following two crosslink tracking scenarios are considered:

- Apogee tracking: two hours around each apogee at 10-second intervals
- Continuous tracking: continuous tracking at 2-minute intervals

Table 4 lists the eight GEONS solutions that were generated and the largest relative position errors among the satellites for each solution. Figure 2 shows the steady-state maximum absolute position errors for the eight solutions. The RMS position errors were approximately 50 to 65% of the maximum errors with these solutions. Relative position errors were computed for the three remote satellites, i.e., satellites 2, 3, and 4, with respect to the local satellite 1. Relative errors for remote-to-remote satellite pairs were computed in a few selected cases. Their results were found to be similar to those of local-to-remote pairs presented in Table 4.

**Table 4. Steady State Relative Position Errors**

| Solution | Description | Relative Position Error (km) | |
|---|---|---|---|
| | | RMS | Maximum |
| 1 | GS Doppler | 3.1509 | 5.789 |
| 2 | GS range | 1.3418 | 2.317 |
| 3 | GS Doppler and GS range | 0.5488 | 0.932 |
| $4^{+}$ | Crosslink range | 0.1257 | 0.733 |
| 5 | GS Doppler, GS range, and crosslink range (apogee tracking) | 0.0438 | 0.129 |
| 6 | GS Doppler and crosslink range (apogee tracking) | 0.0668 | 0.166 |
| 7 | GS Doppler and crosslink range (continuous tracking) | 0.0134 | 0.031 |
| 8 | 2-Day GS Doppler plus 5-day crosslink range (continuous tracking) | 0.0023 | 0.015 |

$^{+}$ Initial relative position errors for all pairs of satellites (at apogee) are < 9 kilometers.

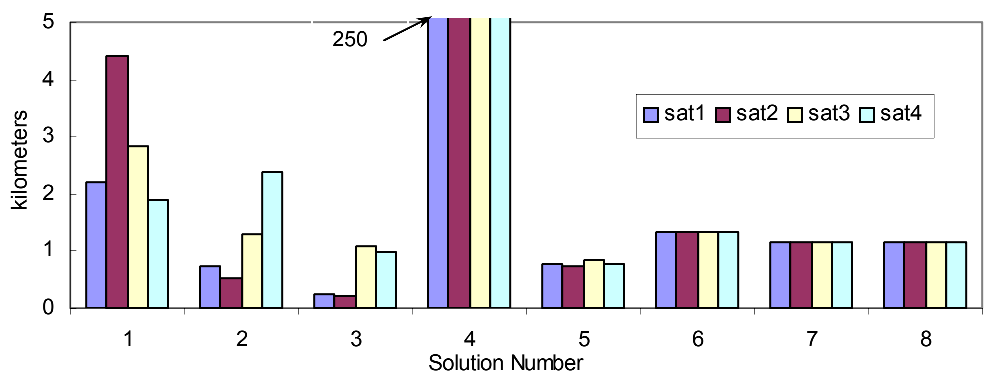


**Figure 2. Steady-State Maximum Absolute Position Errors**

*Solutions Obtained Using GS Data Only*

The first three solutions shown in Figure 2 were obtained using only GS data. The first solution, which uses only Doppler data, easily satisfies the science-driven absolute position accuracy requirements of 100 kilometers, but the relative position errors are too large. The second solution, which uses only GS range data, has position errors that are generally smaller than the corresponding errors from GS Doppler-only solution. Position errors associated with the third solution, which used both GS Doppler and range data, were further reduced. However, for all these solutions, the relative position errors are greater than 1% separation distance for most of the time.

The Doppler and range data used for these solutions have low random noise of 7 mHz-S and 2 meters (1 sigma), respectively. When a range bias of 15 meters was added to the GS range data, the overall quality of the GEONS filter solution did not change in terms of absolute and relative position errors. However, when the GS Doppler data noise was increased from 7 to 35 mHz-S, the relative position errors increased by a factor of 2 to 3. The absolute position errors did not increase as much.

*Solutions Obtained Using Only Crosslink Data*

Solution 4 in Figure 2 exhibits several properties characteristic of solutions using only the crosslink range data. The absolute position errors did not reduce below a few hundred kilometers, much larger than the science-driven absolute position accuracy requirement of 100 kilometers. However, even with such large absolute position errors, the relative position errors, shown in Figure 3, meet the science accuracy goal of 1% of the inter-satellite separation. The filter convergence was found to be more sensitive to the initial relative position errors than to the initial absolute position errors. In addition, all member satellites in the formation have similar absolute position errors, which was not the case with the first three solutions. Although the solution using only the crosslink range data has large absolute position errors, these errors are evenly distributed among all member satellites producing small relative errors.

The initial absolute position errors assumed for Solution 4 in Figure 2 range from 9 to 17 kilometers, while the corresponding initial relative errors range from 1.7 to 8.7 kilometers. The epoch of the solutions was chosen to be one of the apogees, where the inter-satellite separation is approximately 10 kilometers or less. Thus, the initial errors assumed for Solution 4 are conservative. Several other solutions obtained using different initial conditions with the same orders of magnitudes all produced similar solutions. Starting the filter with initial relative position errors larger than 10 kilometers resulted in unstable or divergent solutions in a number of cases.

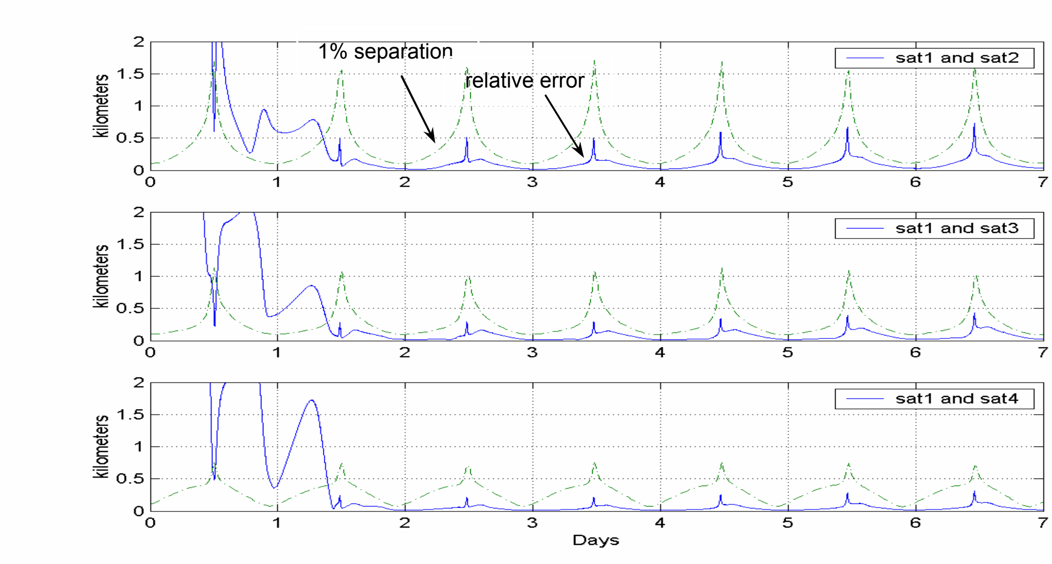


**Figure 3. Relative Position Errors and 1% of Inter-Satellite Separation Solutions Obtained Using Crosslink Data Only (Solution 4 in Table 4)**

The crosslink range data used for Solution 4 was simulated with a random noise of 2 meters (1 sigma). The noise in the crosslink range data is known to increase as the inter-satellite separation distance increases. The 2-meter (1 sigma) noise may be appropriate near apogees, but too small near perigees where the inter-satellite separation may increase to approximately 16 times the separation at apogees. To investigate the sensitivity of the crosslink range only solutions to the noise level in the crosslink data simulation, two additional sets of crosslink range data were generated with a larger random noise sigma of 32 meters ($1\sigma$), one at 2-minute intervals and one at 6-minute intervals. GEONS filter solutions obtained using these two sets of crosslink range data were found to be similar to Solution 4 (shown in Figures 2 and 3) in terms of absolute and relative position errors. When using the 6-minute data, the crosslink range noise root-variance specified for the GEONS filter had to be reduced to 250 meters from 500 meters. Solutions obtained using only crosslink range measurements were particularly sensitive to the initial relative position errors, measurement noise levels, and the measurement noise root-variance specified for the filter, indicative of marginal stability.

*Solutions Using GS and Crosslink Measurements*

Solutions 5 through 8 were obtained using combinations of GS and crosslink data. These solutions exhibit the positive characteristics of the GS data only solutions, i.e., small absolute position errors, and of the crosslink range only solutions, i.e., absolute errors evenly distributed among satellites. Relative position errors derived from these solutions are significantly smaller than those associated with the first four solutions in Table 4. However, the filter's performance is sensitive to the processing start time for the crosslink range data. In addition, the filter was sensitive to the measurement noise root-variances specified for GS and crosslink data. It was difficult to obtain converged filter solutions by processing both GS and crosslink data from the beginning of the data span. In general, more stable filter solutions were obtained by processing GS data from the beginning, and introducing crosslink data a day or two later, when the position errors, especially the relative position errors, are reduced to a level low enough for the crosslink range data to be effective. When the crosslink data is introduced, the solution evenly distributes the absolute errors among the member satellites and reduces the overall relative errors while maintaining the level of absolute errors achieved by GS data.

Solutions 5, 6, and 8 were obtained by processing only GS data for the first 2 days, and then only crosslink data for the remainder of the data span. Solution 7 was obtained by processing only GS data for the first 2 days, and then both GS and crosslink data for the remainder of the data span. These four solutions have similar position error characteristics. The resultant relative errors are at least an order of magnitude smaller than those derived using only GS data. The solutions obtained using GS Doppler plus continuous crosslink range data (the last 2 solutions) meet the absolute and relative accuracy goal of 100 kilometers and 100 meters, respectively. In terms of position errors, these two solutions are of similar quality.

Solution 5 is better than Solution 6 in terms of overall position error. The addition of GS range data produces a more noticeable improvement in the absence of crosslink data (compare Solutions 1 and 3). The relative errors for Solution 8 are somewhat lower than those of Solution 7. However, the absolute velocity errors for Solution 7 are an order of magnitude smaller than those of Solution 8 near a perigee. Thus, both GS Doppler and crosslink data are needed to get more accurate absolute velocities near perigee where many maneuvers will occur.

It is possible to reduce the relative position errors for Solution 7 to the same level as that of Solution 8 by increasing the GS Doppler measurement standard deviation when the crosslink range data are introduced (after processing only GS Doppler data for 2 days). When the Doppler standard deviation was increased from 0.05 to 0.5 Hertz after 2 days, the largest RMS relative position error was reduced from 13.4 to 5 meters, which is closer to the corresponding RMS relative error of 2.3 meters for Solution 8 given in Table 4. However, the absolute velocity errors near perigee for this new solution increased to approximately three times those for the original Solution 7. Comparing with Solution 8, this new solution gives comparable relative position errors and somewhat better velocity errors. Therefore, if the GS Doppler data is available, it should be processed together with crosslink data.

One of the characteristic properties of the solutions obtained using GS data or GS plus crosslink data is that the filter root-sum-variance (RSV) is much larger (by 5 to 10 times) than the state errors based on the estimated and truth state differences. Attempts to reduce filter RSV usually lead to larger state errors. Further filter tuning may be needed to achieve solutions with more realistic filter RSV.

*Relative Semimajor Axis Errors*

Table 5 summarizes the relative semimajor axis (SMA) error statistics derived from the same solutions presented in Figure 2. The error statistics (RMS and maximum values) are taken over an angular (true anomaly) interval of 20º about the apogees and perigees. This angular interval corresponds to a time interval of approximately 7.4 hours around apogee, and the same angular spread around perigee corresponds to approximately a 5-minute time interval. The relative position errors are computed for the three remote satellites, i.e., satellite 2, 3, and 4, with respect to local satellite 1. Table 5 lists the largest errors among the three for each solution. These relative SMA errors are similar in magnitude, except for the crosslink only solution (Solution 4), in which the errors are an order of magnitude larger than those in the other cases. The last solution gives the smallest relative SMA errors around apogees.

*Sensitivity of Solutions to Measurement Noise*

The nominal 1-sigma measurement noise levels (7 mHz-S for GS Doppler and 2 meters for crosslink range) assumed for the solutions presented in Figure 2 may represent an optimistic tracking scenario. To assess the sensitivity of the solutions to the measurement noise level, Solution 8 in Figure 2 was further studied with higher measurement noise values. Parameters varied in these cases were GS Doppler noise, crosslink range noise, and the initial time period of processing only Doppler data. The Doppler noise values used are 7 and 35 mHz, crosslink range noise of 2, 50, and 100 meters, and the initial GS Doppler processing period of 1 and 2 days. In the six cases examined, the maximum absolute errors were below 10 kilometers.

**Table 5. Steady State Relative SMA Errors (Meters)**

| Solution | Description | (Apogee –10°, Apogee +10°) | | (Perigee –10°, Perigee +10°) | |
|---|---|---|---|---|---|
| | | RMS | Maximum | RMS | Maximum |
| 1 | GS Doppler | 8.827 | 15.494 | 44.234 | 80.821 |
| 2 | GS range | 18.157 | 39.776 | 42.593 | 65.015 |
| 3 | GS Doppler and GS range | 4.379 | 13.814 | 41.938 | 77.306 |
| 4 | Crosslink range | 15.216 | 35.754 | 238.126 | 374.795 |
| 5 | GS Doppler, GS range, and crosslink range (apogee tracking) | 11.140 | 24.034 | 42.279 | 74.155 |
| 6 | GS Doppler and crosslink range (apogee tracking) | 8.712 | 18.675 | 42.137 | 82.651 |
| 7 | GS Doppler and crosslink range (continuous tracking) | 5.926 | 15.597 | 40.485 | 76.532 |
| 8 | 2-Day GS Doppler plus 5-day crosslink range (continuous tracking) | 0.793 | 2.849 | 40.982 | 71.625 |

The six cases simulated are labeled with the Doppler and crosslink noise values: (7 mHz, 2 meters), (7 mHz, 50 meters), (7 mHz, 100 meters), (35 mHz, 2 meters), (35 mHz, 50 meters), (35 mHz, 100 meters). A 1-day initial GS Doppler processing interval was adequate for the case of GS Doppler with 7 mHz noise, but for the case of 35 mHz noise, 2 days of initial Doppler data processing generally led to more stable solutions. Therefore, a 1-day initial Doppler processing interval was used for the first three solutions, and 2-day initial Doppler processing interval for the last three solutions. The relative error statistics for the steady state solutions are shown in Figure 4.

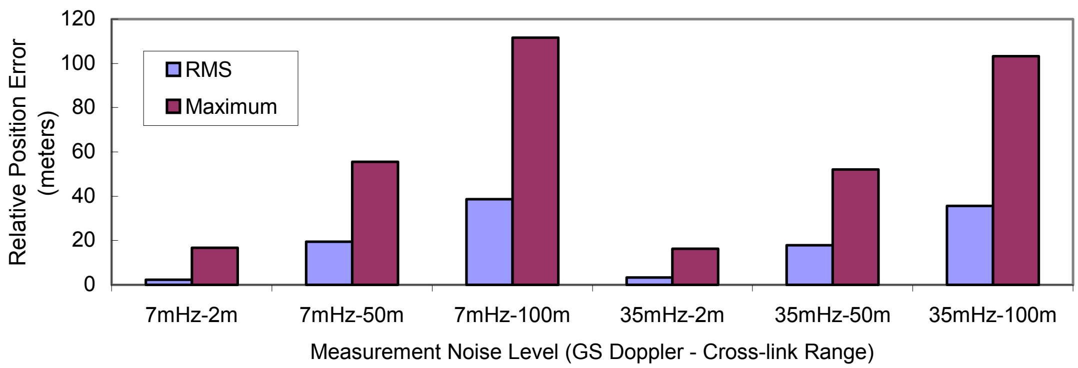


**Figure 4. Dependence of Relative Position Errors on Measurement Noise Level**

These results show a strong dependence of the relative errors on the crosslink range noise: the higher the crosslink noise, the larger the relative errors. It should be noted, however, that the results shown in Figure 4 were obtained using the same set of filter tuning parameters for all solutions. By retuning the filter for higher crosslink measurements noise, the sensitivity of the relative errors to the crosslink range noise could be reduced. All six solutions in Figure 4 are seen to be acceptable in terms of science-derived relative error requirements. Their absolute position errors are well within 100-kilometer science requirement as mentioned earlier. Thus, the estimation scenario involving the initial Doppler data processing and then switching to the crosslink range data processing is an acceptable concept for the MMS Phase 1 navigation support.

*Effects of Gravity Modeling and Measurement Rate*

The GEONS filter solutions discussed above were obtained using the filter parameter values listed in Table 2. Using different values for these parameters will give different results. For a few selected cases, different force model and measurement processing options were tested. For example, using the truncated Earth gravity model JGM2 8x8 instead of JGM2 4x4, the absolute position errors changed by approximately 20%, but the relative position errors remained unchanged. The results obtained using the force model with and without extra planets produced similar results. In another example, the continuous crosslink range data rate was changed from one every 2 minutes to one every 6 minutes. The 6-minute results were found to be essentially the same as the 2-minute results. In some cases, especially in the case with a higher crosslink range noise, the measurement noise root-variance (specified for GEONS filter) may have to be readjusted (for example, reducing it from 500 to 250 meters). The first and fifth solutions shown in Figure 4 are studied further using Monte Carlo (MC) simulations in Section 2.4.

## 3 – SOLUTIONS OBTAINED USING GPS PSEUDORANGE AND CROSSLINK MEASUREMENTS

The following combination solutions are studied:

- GPS pseudorange (PR)
- GPS PR and singly differenced GPS PR (SDPR)
- GPS PR and local (remote-to-local) crosslink range
- GPS PR and local and remote (remote-to-remote) crosslink range (for 10 minute every hour at a rate of one set of measurements per minute)
- GPS PR for local satellite and crosslink range from all remote satellites (not processing GPS PR's from remote satellites)

The GPS signal acquisition threshold was 31 dB-Hertz, and the crosslink range noise was 2 meters. Figure 5 shows the number of GPS SVs that can be acquired and tracked by a receiver from MMS Phase 1 orbits, assuming a signal acquisition threshold of 31 dB-Hertz. With this reduced threshold receiver, the GPS SV visibility increases by approximately 60% over that of a typical 35-dB-Hz receiver.

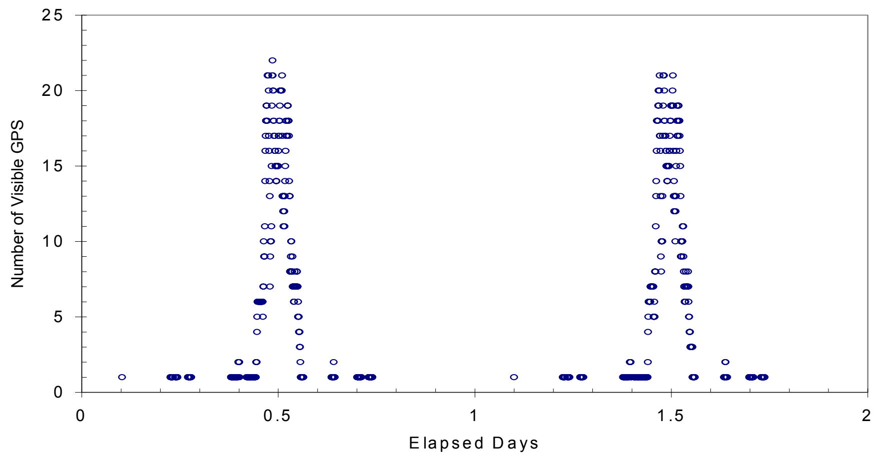


**Figure 5. GPS SV Visibility as a Function of Time**

Both absolute and relative state estimation modes were examined. In relative state estimation mode, the relative state vectors of the remote satellites with respect to the local satellite are directly estimated together with the absolute state vector of the local satellite. Table 6 shows the absolute and relative position errors associated with these solutions. The local crosslink tracking data began 12 hours into the simulation span, which permitted enough GPS data to be processed to stabilize the filter. The crosslink range measurements are two-way without any range biases. In general, more accurate absolute solutions were obtained when estimating absolute state vectors, and more accurate relative solutions were obtained when estimating the relative state vectors. The addition of crosslink range data improved the relative solutions.

Table 7 summarizes several cases that were made processing GPS and both local and remote crosslink range data for various GPS acquisition thresholds. The crosslink range noise was set at 2 meters. Relative state estimation mode was used, and the first crosslink pass began 12-hours into the data span. No attempt was made to retune the filter for each case in order to obtain a "better" solution. The absolute position errors are quite similar for all cases examined, gradually decreasing as the acquisition threshold was reduced. The relative position errors are quite similar using acquisition thresholds ranging from 28 to 35 dB-Hertz and crosslink tracking data, and increase significantly for the 20-dB-Hertz (with higher noise) case and the 35-dB-Hertz case without crosslink data.

**Table 6. Steady State Absolute and Relative Position Errors: GPS and Crosslink**

| Tracking Scenario | Estimation Mode | Absolute (meters) | | Relative (meters) | |
|---|---|---|---|---|---|
| | | RMS | Maximum | RMS | Maximum |
| GPS-only | Absolute | 61 | 189 | 10 | 57 |
| | Relative | 83 | 254 | 3 | 15 |
| GPS + SDPR | Absolute | 43 | 145 | 20 | 76 |
| | Relative | 60 | 198 | 3.6 | 27 |
| GPS+crosslink (local only) | Absolute | 88 | 264 | 2.3 | 11 |
| | Relative | 131 | 384 | 1.8 | 6.5 |
| GPS+crosslink (local+remote) | Absolute | 58 | 176 | 2 | 8.3 |
| | Relative | 85 | 253 | 2.9 | 5.5 |

**Table 7. Steady State Absolute and Relative Position Errors for Various GPS Signal Acquisition Thresholds: GPS and Crosslink**

| Case | Absolute (meters) | | Relative (meters) | | GPS Visibility (percent) |
|---|---|---|---|---|---|
| | RMS | Maximum | RMS | Maximum | |
| 20 dB-Hertz | 60 | 169 | 5.8 | 40 | 88 |
| 28 dB-Hertz | 64 | 194 | 1.6 | 5.3 | 43 |
| 30 dB-Hertz | 76 | 241 | 1.7 | 5.3 | 27 |
| 31 dB-Hertz | 84 | 253 | 1.4 | 4.0 | 24 |
| 33 dB-Hertz | 78 | 262 | 1.0 | 3.5 | 19 |
| 35 dB-Hertz | 76 | 256 | 1.3 | 4.0 | 14 |
| 35 dB-Hertz (no crosslink) | 76 | 257 | 2.3 | 10 | 14 |
| 31 dB-Hertz (GPS PR on local satellite only) | 130 | 513 | 21.8 | 63.5 | 24 |

The last solution in Table 7 was obtained using GPS PR measurements for the local satellite and all crosslink range data from remote satellites (i.e., GPS PR data from remote satellites are not included in the solution). In this scenario, the GPS data on the local satellite is expected to fix the local satellite absolute position, and the crosslink data provides the relative positions for the remote satellites, which, in turn, can provide the absolute positions for the remote satellites as well. The results listed in Table 7 are the largest absolute and relative errors among the four satellites in the formation, and show that they are well within the science-driven position accuracy requirements.

The GPS measurements were simulated assuming a high quality GPS receiver clock close to a Rubidium standard. In a previous study MMS Phase 1 orbits (ref. 6), it was shown that the performance of a 30 dB-Hertz receiver with a temperature controlled crystal oscillator (TCXO)-quality clock was very similar to that of a Rubidium-quality clock in terms of the absolute and relative position errors. Even if the absolute and relative errors increase by an order of magnitude when using a GPS receiver clock of TCXO quality, they would still meet the stated absolute and relative science accuracy goals of the MMS Phase 1 mission. However, to be on the safe side, a reduced signal acquisition threshold receiver should be used if a TCXO quality receiver clock is used.

Semimajor axis errors were computed for the GPS solutions obtained with and without crosslink range measurements using a signal acquisition threshold of 35 dB-Hertz. The contribution from crosslink data to the SMA errors in this case appears to be insignificant. The SMA errors from these solutions are similar to those from solutions obtained using GS Doppler plus crosslink data.

## 4 - MONTE CARLO SIMULATIONS

Monte Carlo (MC) simulations were made for the following cases:

- GPS pseudorange (31-dB-Hertz) plus crosslink (remote and local) range
- GS Doppler plus crosslink range (nominal noise levels)
- GS Doppler plus crosslink range (higher noise levels)

The measurement noise levels assumed for the cases of GS Doppler plus crosslink range data are:

- Nominal noise levels: 7 mHz for Doppler and 2 meters for crosslink range
- Higher noise levels: 35 mHz for GS Doppler and 50 meters for crosslink range.

The MC simulations were performed to assess not only definitive OD statistics, but also to determine how the orbit errors associated with a long-term propagation grow with time. The propagation information is needed for formation control and maneuver planning purposes. The MC simulations used a definitive span of six days and a predictive span of 36 days. Statistics were generated over about 25 Monte Carlo simulations. The statistics include definitive, predictive, and 6-hours around the apogee events. The pseudo-random number seeds for the measurement noise and receiver clock and GPS ephemeris errors (GPS solutions only) were varied for each run. Tables 8 through 10 summarize the Monte-Carlo statistics.

**Table 8. Steady State Absolute and Relative Position and SMA Errors Processing GPS PR (31 dB-Hertz) and Crosslink Range**

| Error Type | Six-Day Definitive (km) | | 36-Day Predictive[1] (km) | 36-Day Predictive (apogee)[2] (km) |
|---|---|---|---|---|
| | RMS | Maximum | Maximum | Maximum |
| Absolute position | 0.100 | 0.362 | 111 | 29 |
| Absolute SMA | 0.046 | 0.526 | 1.7 | 0.15 |
| Relative position | 0.0014 | 0.007 | 1.5 | 0.33 |
| Relative SMA | 0.004 | 0.061 | 0.14 | 0.002 |

[1]Statistics taken around entire orbit [2]Statistics taken six hours around apogee

**Table 9. Steady State Absolute and Relative Position and SMA Errors Processing GS Doppler and Crosslink Range (Nominal Noise Levels)**

| Error Type | Six-Day Definitive (km) | | 36-Day Predictive[1] (km) | 36-Day Predictive (apogee)[2] (km) |
|---|---|---|---|---|
| | RMS | Maximum | Maximum | Maximum |
| Absolute position | 1.2 | 4.0 | 151 | 39 |
| Absolute SMA | 0.037 | 0.524 | 2.2 | 0.19 |
| Relative position | 0.0025 | 0.027 | 2.0 | 0.36 |
| Relative SMA | 0.004 | 0.065 | 0.17 | 0.002 |

[1]Statistics taken around entire orbit [2]Statistics taken six hours around apogee

**Table 10. Absolute and Relative Position Errors Processing GS Doppler and Crosslink Range (Higher Noise Levels)**

| Error Type | Six-Day Definitive (km) | | 36-Day Predictive[1] (km) | 36-Day Predictive (apogee)[2] (km) |
|---|---|---|---|---|
| | RMS | Maximum | Maximum | Maximum |
| Absolute position | 3.7 | 12 | 197 | 52 |
| Absolute SMA | 0.056 | 0.679 | 2.4 | 0.24 |
| Relative position | 0.020 | 0.073 | 14 | 3.7 |
| Relative SMA | 0.014 | 0.11 | 0.25 | 0.014 |

[1]Statistics taken around entire orbit [2]Statistics taken six hours around apogee

The MC simulation results are similar to the corresponding results based on single solution simulations discussed in Sections 2 and 3. The absolute position errors of the single solution simulations are somewhat smaller than the corresponding time-wise ensemble statistics of MC solutions. The maximum absolute position errors derived from the MC simulation are still well within the 100-kilometer science requirement. In the case of the relative position errors, the two simulation results are very similar, with the MC maximum relative errors being somewhat larger than the corresponding single solution results.

The solutions obtained using GS Doppler and crosslink range measurements (nominal noise case) give absolute position errors approximately 10 times larger than the corresponding solutions obtained using GPS PR and crosslink range measurements. However, the relative position errors from the solutions obtained using GS Doppler and crosslink range data are only 2 to 4 times larger than those from the corresponding solutions obtained using GPS PR and crosslink range data. The differences in the absolute velocity errors are larger between solutions obtained using GPS PR and crosslink range data and those obtained using GS Doppler data. Definitive velocity errors associated with the solutions obtained using GPS PR and crosslink data are on the order of 0.004 meters/second near apogees (6-hour RMS) and 0.03 meters/second near perigees (maximum around perigees). The corresponding velocity errors associated with the solutions obtained using GS Doppler and crosslink data (with nominal noise) are on the order of 0.05 meters/second and 1.5 meters/second, respectively. In the case of GPS PR and crosslink data, solutions were obtained by processing a large amount of GPS data around perigees (approximately 20 GPSs visible to MMS

Phase 1 satellites in the vicinity of perigees), leading to states more accurately determined than in the case of GS Doppler plus crosslink data.

The solutions obtained using the GS Doppler and crosslink data were obtained by processing GS Doppler data for the first 2 days and only crosslink data for the remainder of the data span. As discussed earlier, processing GS Doppler data together with the crosslink data throughout the data span could reduce velocity errors around the perigee by almost an order of magnitude. To support formation control maneuvers, more accurate velocity solutions may be required. Such solutions can be obtained by processing an increased amount of the GS Doppler data throughout the data span together with the crosslink data.

Figures 6 and 7 show the time variations of the absolute position and SMA errors for satellite 4 around apogees (apogee -3.5 hours to apogee +3.5 hours). Errors for the other satellites show similar behaviors. Absolute position errors at perigees are 8 to 10 times larger than those at apogees. The GS Doppler and crosslink solutions with higher noise values give similar mean errors, but much larger maximum errors than the GS Doppler and crosslink solutions with nominal noise values.

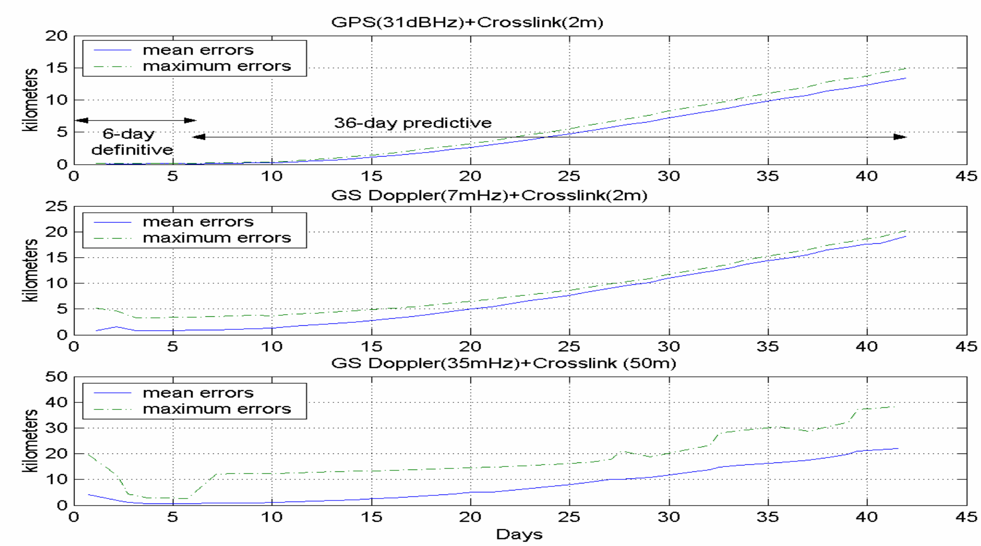


**Figure 6. Ensemble Mean and Maximum Absolute Position Errors Near Apogee**

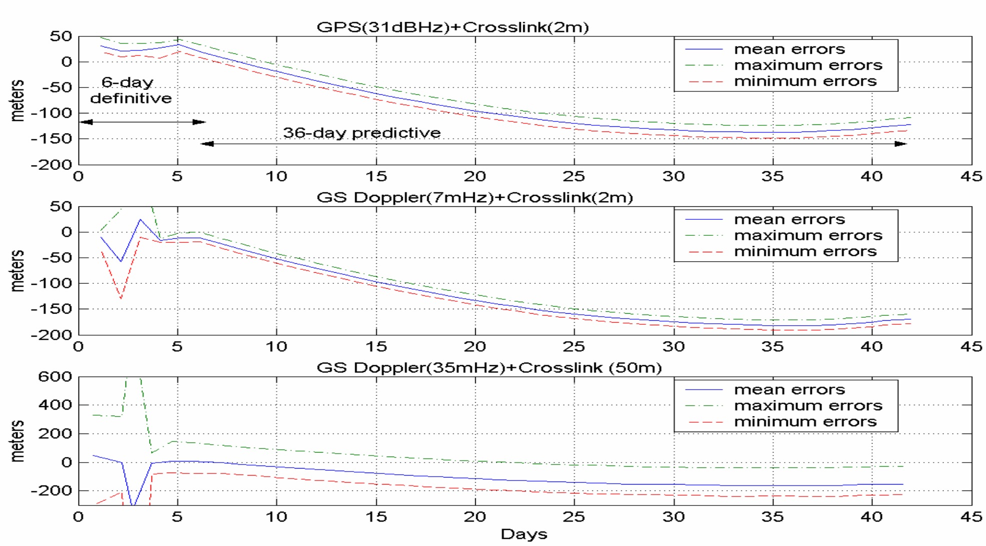


**Figure 7. Ensemble Mean and Maximum Absolute SMA Errors Near Apogee**

The maximum absolute position prediction errors occur near perigee. The maximum predicted position error reaches 100 kilometers in approximately 33 days for solutions using GPS and crosslink data and in 27 days for solutions using GS Doppler and crosslink (nominal noise) (ref. 2). In the case of GS Doppler plus crosslink with larger noise, the maximum absolute errors reach 100 kilometers in approximately 23 days. The 36-day predicted absolute position errors near apogees are all less than 40 kilometers. The predicted absolute position errors, in most cases, are predominantly in the in-track direction. The magnitudes of the absolute SMA errors near apogees vary from 100 to 250 meters. Note that the difference between the Earth gravity models used in the filter and the truth trajectory (i.e., 4x4 versus 8x8) is also a major contributor to the absolute prediction error.

Similar results for the relative errors are shown in Figures 8 and 9. The 36-day predicted ensemble mean and maximum relative position errors near apogees for the two cases with the nominal noise values (GPS plus crosslink and GS Doppler and crosslink) are very similar, with maximum errors of less than 200 meters. In terms of the ensemble mean relative position errors, the solutions using GS Doppler and crosslink data are somewhat better, but the solutions using GPS and crosslink are better in terms of the ensemble maximum errors. The corresponding errors for the case of GS Doppler and crosslink data with larger noise values reach 2.5 kilometers in 36 days. Thus, with the first two navigation scenarios with nominal noise levels, the accuracy of the relative positions of the satellites in the formation can be maintained to within 200 meters in propagation mode for up to 5 weeks. However, in the case of GS Doppler and crosslink data with higher noise values, the maximum relative position error reaches 200 meters in less than 10 days. As in the case of the absolute position errors, the relative position errors are also dominated by in-track errors, but to a lesser degree. Occasionally, radial and cross-track errors are quite sizable in magnitude, almost comparable to the magnitude of the in-track errors. The in-track error growth rate (both absolute and relative) is proportional to the SMA errors in a two-body approximation (ref. 7). In the case of MMS Phase 1 orbits, this approximate relationship between in-track errors and SMA errors holds fairly well at apogee.

The results of definitive and predictive errors discussed in this section will be needed for formation maintenance and control maneuver planning and validation purposes. One of the factors that will be used to determine when such maneuvers have to be performed will be the actual deviation of the relative distance from a prescribed value (relative control box). If the member satellites need to maintain the relative distances around apogees to within 1 kilometer with respect to the prescribed nominal separation distances, the current or future separation distance obtained from a definitive or a predictive orbit solution should be known to within a fraction of this 1-kilometer tolerance. In this example, a 200-meter maximum relative orbit accuracy may be considered adequate. The solutions obtained using GS Doppler and crosslink range with higher noise may not be adequate for long-term maneuver planning. The solutions obtained using the nominal noise levels, on the other hand, can be used to perform a long-term maneuver analysis, since these solutions can predict the future formation behaviors accurately for more than a month.

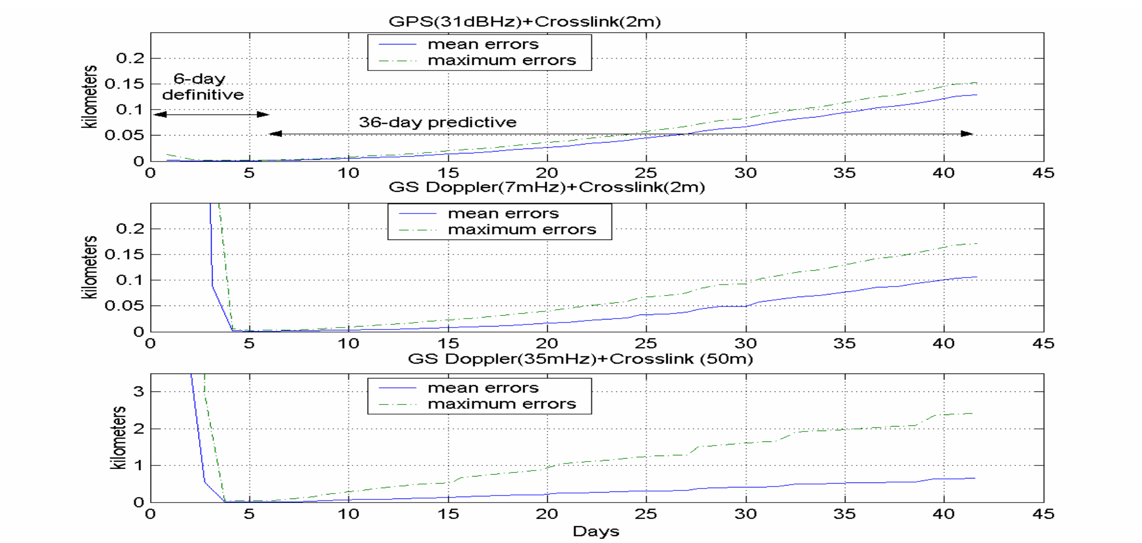


**Figure 8. Ensemble Mean and Maximum Relative Position Errors Near Apogee**

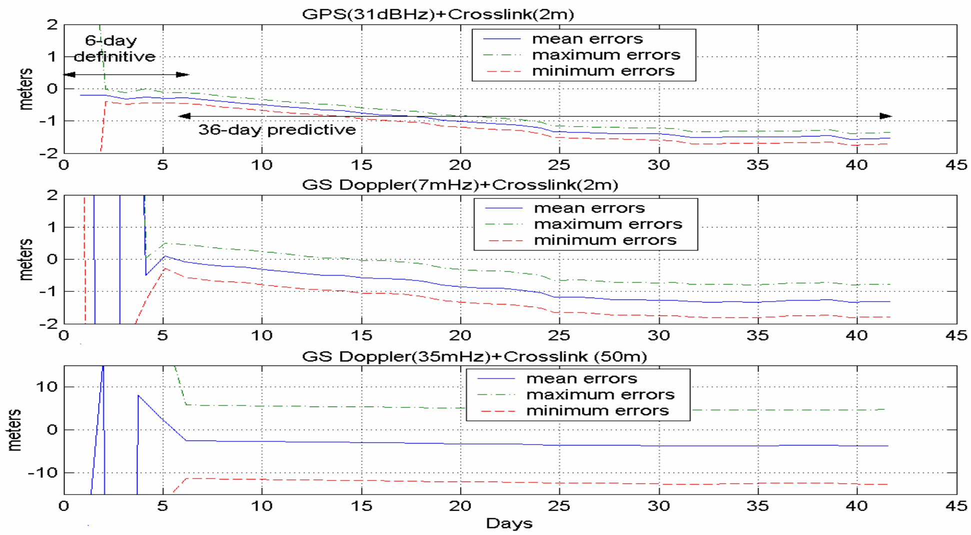


**Figure 9. Ensemble Mean and Maximum Relative SMA Errors Near Apogee**

## 5 – CONCLUDING REMARKS

A number of different navigation solution strategies for MMS Phase 1 orbits were evaluated using GEONS to process simulated tracking measurements. The types of tracking measurements simulated were GS Doppler measurements, GS range measurements, inter-satellite crosslink range measurements, and GPS pseudorange measurements. The GS data simulation assumed three 20-minute contacts per day per satellite. The crosslink range data were simulated at a rate of one observation every 2 minutes. In the case of GPS pseudorange measurements, all available measurements were simulated. Navigation solutions were obtained using the GEONS filter and various combinations of these measurements types.

The main conclusions derived from this study are listed below. The results from various OD approaches are summarized in terms of whether they meet the science objectives. The science-driven navigation accuracy requirements for this mission are post-processing knowledge of the absolute spacecraft position to within 100 kilometers and knowledge of the inter-spacecraft ranges (relative positions) to within 1% of the actual separation (100 meters around apogees at 10-kilometer separations).

*Definitive Orbit Determination Accuracy*

It appears that the most promising navigation options for the MMS Phase 1 mission orbits are (1) ground navigation using GS Doppler and crosslink range data, (2) onboard navigation using GPS measurements, (3) onboard navigation using GPS and crosslink measurements, and (4) onboard navigation using GPS measurements on the local satellite only and all crosslink range measurements from remote satellites. Option 1 requires downlinking the crosslink measurements to the GSs. Options 2, 3 and 4 need GPS receivers onboard. All of these options can meet both the absolute and the relative accuracy requirements.

For the MMS Phase 1 mission orbits, options that use GPS measurements (Option 2 or 3) will be operationally simple and provide more accurate and reliable navigation support than any other scenarios examined in this report. Option 3 will generally give a better relative accuracy than Option 2. If all crosslink range data are available to all satellites in the formation on a continual basis, Option 4 will be equally attractive, and may be the simplest to use, although somewhat less accurate compared with Options 2 and 3. An attractive feature of using Option 2 is that each member satellite can generate its own independent single satellite absolute state solution and transmit these state vectors to other members in the formation at given intervals, thereby alleviating the need of transferring measurement data between satellites. In this case, the relative states can be computed by differencing these absolute states.

*Predicted Orbits Accuracy*

To study the long-term propagation properties of MMS Phase 1 orbits, 36-day predicted orbits were generated from the Monte Carlo simulation solutions for three different tracking scenarios with different measurement noise levels.

With noise values close to the nominal values used in this study, the predicted maximum absolute position errors grow to be 150 kilometers, and maximum relative position errors around apogees grow to 200 meters over 36 days. However, with larger noise values (35 mHz-S for the GS Doppler and 50 meters for the crosslink range), the predicted maximum absolute position errors reach 200 kilometers, and maximum relative position errors around apogees reach 2.5 kilometers over 36 days.

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